Optimization of Semi-Passively Actuated Mechatronic Systems

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Abstract

A mechatronic system as a general controlled multibody system is under the consideration. In the paper special emphasizes are put on the study of underactuated and overactuated systems having different type of actuators (external powered drives, unpowered spring-damper like drives, etc.). The mathematical statement of the optimal control problem is proposed that is suitable for modeling of controlled motion and optimization of mechatronic systems with different type and degree of actuation. The methodology and the numerical algorithm for solving of control and optimization problem for the semi-passively actuated mechatronic systems are presented. Solutions of several optimal control problems for semi-passively actuated mechatronic systems are demonstrated, (the energy-optimal control of closed-loop chain semi-passively actuated SCARA-like robot and the problem of optimization of the hydraulic and pneumatic drives of the multibody system modeled the human locomotor apparatus with above-knee prosthesis).

1 Introduction

One of the primary goals of mechatronics is to gain as many advantages as possible from the optimal interaction between mechanical, control, electronic and computer subsystems. This requires more fundamental research on a number of topics of controlled multibody systems, e.g. control-structure interaction, parameter identification and optimal design, contact and impact problems, large deformation problems, etc., [1]. The research in the above areas can help to improve performance characteristics of modern mechatronic products.

The important and relevant characteristics of interaction between inherent dynamics and control of any mechanical system are its degree and type of actuation. Most technical systems usually have the same number of actuators as degrees of freedom of their mechanical subsystems, i.e. they belong to the class of fully actuated mechanical systems. If mechatronic system has fewer actuators than joints or more precisely if the dimension of the configuration space exceeds that of the control input space, the system is called *underactuated.*

The modern mechatronics systems must be autonomous and dexterous. Dexterity implies the mechanical ability to carry out various kinds of tasks in various situations. Systems must have many sensors and more actuators than degrees of freedom, i.e. being the system with sensing and actuation redundancies. Obviously, the type of actuators used can also be different depending on the task of robot.

The analysis of the literature shows the importance of studying dynamics, control and optimization problems of mechatronic systems with different degree and type of actuation and the robotic systems, in particular. This research is of a great challenge.

2 Statement of the Problem

Consider a mechatronic system the controlled motion of which can be described by the following equations:

$$\dot{x} = f(x,u,w(t,\xi)), \quad g(x,w(t,\xi)) = 0, \quad t \in [0,T] \quad (1)$$

Here $x = (x_1, x_2, ..., x_n)$ is a state vector, $u = (u_1, u_2, ..., u_m)$ is a vector of controlling stimuli (forces, torques) generated by the external (powered) drives of the system, $w = (w_1, w_2, ..., w_r)$ is a vector of the controlling stimuli of the internal (unpowered) drives, and T is the duration of the controlled motion. Vector functions f and g are determined by the structures of the system and unpowered drives, respectively, ξ is a vector of design parameters of the unpowered drives.

Constraints and restrictions are imposed on the state vector *x(t)*, the controlling stimuli of the unpowered drives *w(t,ξ)*, and the external control laws *u(t)* of the system. These restrictions can be written in the following way:

$$\{x(t)\} \in Q, \qquad t \in [0, T] \tag{2}$$

$$w(t, \xi) \in W, \qquad t \in [0, T] \tag{3}$$

$$u(t) \in U, \qquad t \in [0, T] \tag{4}$$

In formulas (2) - (4), Q and U are given domains in the state and control spaces of the system, respectively; W is a set of addmissible controlling stimuli determined by the structure of the unpowered drives.

The differential equations (1) together with the restrictions (2)-(4) are called the mathematical model of the semi-passively actuated mechatronic system. This model can be used for many applications, e.g. to study fundamental questions about the role of inherent dynamics in controlled motion, and how much mechatronic system should be governed by the external drives and how much by the system's inherent dynamics; for computer simulation of the energy-optimal motion of closed-loop chain manipulator robots with unpowered drives [2-5], etc.

The following optimal control problem can be formulated.

Problem A. Given a mechatronic system the controlled motion of which is described by equations (1). It is required to determine the vector-function $w_*(t, \xi)$, the motion of the system $x_*(t)$ and the external controlling stimuli $u_*(t, x_*, w_*)$ which alltogether satisfy the equations (1), the restrictions (2)-(4), and which minimize the given objective functional $\Phi[u]$.

As a result of the solution of *Problem A* the optimal structure of mechatronic system having both powered and unpowered drives is designed. The external controlling stimuli for the system are also found which minimize the given objective functional.

3 Methodology

To solve *Problem A* the numerical method has been developed for systems, which model semi-passively actuated manipulator robots and bipedal locomotion robots with unpowered drives at their joints [2-5]. The method is based on a special procedure to convert the initial optimal control problem (*Problem A*) into a standard nonlinear programming problem: $F(C) \Rightarrow \min_C$, $g(C) \le 0$, where C is a vector of varying parameters. This is made by an approximation of the independently varying functions $q(t)$ by a combination of the fifth order polynomial and Fourier series, i.e. by using the following expressions:

$$q(t) = \sum_{j=0}^{5} C_{qj} (t - t_0)^j +$$

$$\sum_{k=1}^{N_q} [a_{qk} \cos(k\omega(t - t_0)) + b_{qk} \sin(k\omega(t - t_0))].$$

Here $\omega = 2\pi / T$, and N_q are given positive integers.

Taking into account the restriction (2), the list of independently varying parameters can be determined. To solve the nonlinear programming problem different algorithms have been used, (algorithms that are based on the Rozenbrock's method, the sequential quadratic programming method, others). The key features of the proposed method for solving *Problem A* are its high numerical efficiency and the possibility to satisfy a lot of restrictions imposed on the phase coordinates of the system automatically and accurately [2-5].

4 Manipulator Robot

Here we present some results of optimization of controlled motion of the semi-passively actuated closed-loop chain SCARA-like robot. The sketch of the robot is shown in Figure 1. Robotic system has the following new features in comparison with the well-known SCARA robot. In addition to powered drives $u_1(t)$, $u_2(t)$ and $u_3(t)$ applied to the links OA, AB, and OD, respectively, it comprises several unpowered (passive) spring-damper-like drives p_1, p_2 and p_3. An additional link OD has also been incorporated into the structure that gives the possibility to obtain the semi-passively actuated closed-loop chain robot.

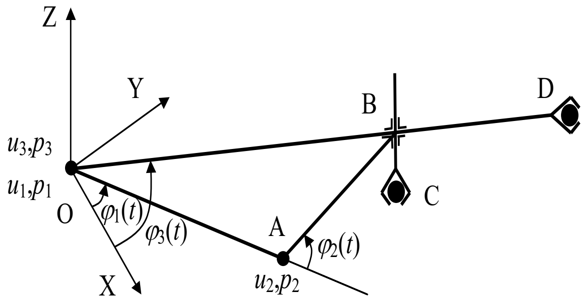

Figure 1. Semi-Passively Actuated Closed-Loop Chain SCARA-Like Robot.

The equations of motion of the considered mechanical system can be derived using the Lagrange formalism and written as follows [3]:

$$f_1(\phi_i, \dot{\phi}_i, \ddot{\phi}_i) = u_1 + p_1 + u_3 + p_3, \quad (5)$$
$$f_2(\phi_i, \dot{\phi}_i, \ddot{\phi}_i) = u_2 + p_2 + b(\phi_i)(u_3 + p_3)$$

The torques of the unpowered drives are modelled by formulas:

$$p_i = -k_i(\varphi_i - \varphi_{i0k}) - c_i\dot{\varphi}_i \quad i = 1,2,3, \quad (6)$$

where k_i are the stiffness coefficients, c_i are the damping coefficients, φ_{i0k} are the no-load angles of the torsional spring.

The considered robot is an overactuated mechatronic system. This makes it possible to optimize the controlling stimuli of powered drives for an arbitrary given motion of the robot.

Problem A.1. Assume that arbitrary motion of the robot and control torques of unpowered drives are given, i.e. the functions $\phi_i(t), p_i(t)$ are specified. It is required to find the control $u = (u_1, u_2, u_3)$, which minimize the functional $E[u(t)] = \int_0^T (u_1^2(t) + u_2^2(t) + u_3^2(t))dt$ subject to the differential constraints (5), (6).

It can be shown that the solution of *Problem A.1* is given by the following formulas:

$$u_3^*(t) = (g_1 + bg_2)/(2 + b^2), \quad u_1^*(t) = g_1 - u_3^*(t) \quad (7)$$
$$u_2^*(t) = g_2 - bu_3^*(t)$$

Here the functions g_1 and g_2 have the expressions:

$$g_1 = f_1 - p_1 - p_3, \quad g_2 = f_2 - p_2 - bp_3 \quad (8)$$

The obtained controlling stimuli (7), (8) provide execution of an arbitrary given motion of the overactuated robot with minimal energy consumption E^*.

The simplest way to reduce the overactuation of the considered robot is to exclude one of the powered drives. For instance, assuming that $u_3(t) = 0, \ t \in [0, T]$, the unique solution of *Problem A.1* for the functions $u_1(t), \ u_2(t)$ can be obtained from the equations (5). In this case the functional E is equal to

$$E^0 = \int_0^T (g_1^2(t) + g_2^2(t))dt \quad (9)$$

where the functions $g_1(t), \ g_2(t)$ are given by the formulas (8).

Comparing the value E^0 with the value of the functional E for the obtained optimal controlling stimuli $u_i^*(t)$ it is easy to show the validity of the following expression

$$E^0 - E^* = \int_0^T (g_1(t) + bg_2(t))^2 / (2 + b^2)dt \quad (10)$$

The formula (10) shows that the energy consumption needed to execute an arbitrary given motion by the considered overactuated robot with obtained optimal controlling stimuli (7), (8) is less than the energy consumption of the same robot but without powered drive acting on the link OD.

Some other results of numerical solution of dynamics, control and optimization problems of semi-passively actuated closed-loop chain SCARA-like robot with different degrees and types of actuation can be found in [3, 5].

5 Bipedal Locomotion System

Here the application of methodology of optimization of semi-passively actuated mechatronic system described in paragraph 3 is demonstrated for solving the design problem of lower limb prostheses.

The amputee locomotor system (ALS) with above-knee prosthesis is depicted in *Figure 2.* ALS is modeled as the mechanical system of seven rigid bodies connected by ideal cylindrical hinges. It is assumed that the control torques $q_i(t), u_i(t), p_i(t)$ acting at the hip (point H), knee (point K_i) and the ankle (point A_i) joints, respectively.

As generalized coordinates that determined the position of the given mechanical system we chose the following: x and y, the Cartesian coordinates of the point of attachment of the legs (the point H); $\psi, \alpha_i, \beta_i, \gamma_i$, the angles of deviation of the link HG, HK_i, K_iA_i, and $A_iT_iH_i$, (i=1,2) respectively from vertical (*Figure 2*).

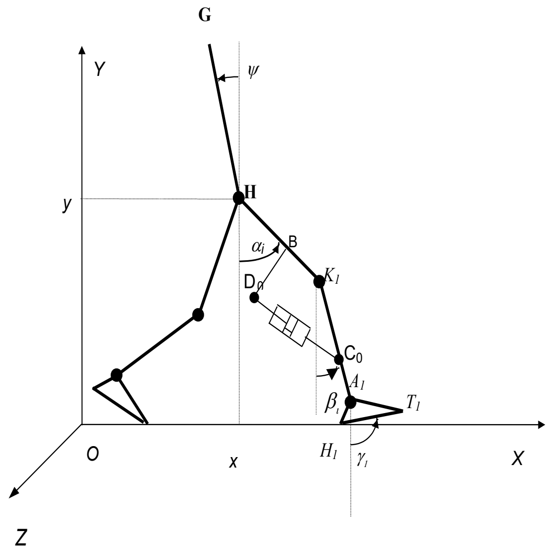

Figure 2. ALS with Above-Knee Prosthesis.

The above-knee prosthesis comprises the linear-viscoelastic ankle mechanism and hydraulic or pneumatic knee mechanism.

During locomotion of ALS with above-knee prosthesis the control torques

$$p_1(t) = C(\beta_1 - \gamma_1 + \pi/2) + K(\dot{\beta}_1 - \dot{\gamma}_1) + D, \quad (11)$$

$$u_1(t) = (P_2 - P_1)S_p d_2 (d_1^2 + l_0^2)^{1/2} \sin(\alpha_1 - \beta_1 + \eta)/l_1$$

are generated at the ankle and at knee joints of the prosthetic leg, respectively.

Here C, K are the torsion spring and the damping coefficients of the ankle mechanism; D is determined by the free angle of the spring and torsion spring coefficients; P_1, P_2 are the chamber pressures of the hydraulic or the pneumatic actuator that can be calculated by using the equations of dynamics of the knee mechanism of the prosthesis, S_p is the cylinder piston cross-area,

$$l_1 = (d_1^2 + d_2^2 + l_0^2 + 2d_2(d_1^2 + l_0^2)^{1/2} \cos(\alpha_1 - \beta_1 + \eta))^{1/2}$$

$$\eta = a\tan(l_0 / d_1),$$

$$d_1 = |BK_1|, \quad d_2 = |K_1C_0|, \quad l_0 = |BD_0|.$$

The detailed description of the considered model of ALS can be found in [2, 6]. The boundary conditions and other constraints on the phase coordinates of the system have been given on the basis of known experimental data on the human gait [7].

The design problem of the above-knee prosthesis can be formulated in the same way as *Problem A* by taken into account that the considered semi-passively actuated mechatronic system has the state vector $\left\{x, \dot{x}, y, \dot{y}, \psi, \dot{\psi}, \alpha_i, \dot{\alpha}_i, \beta_i, \dot{\beta}_i, \gamma_i, \dot{\gamma}_i, i = 1,2\right\}$, the vector of controlling stimuli of the powered drives $u(t) = \{q_1, q_2, u_2, p_2\}$, and the vector of the structural parameters of the unpowered drives $C_p = (C, K, D, d_1, d_2, l_0, S_p, S_0)$.

The following functional

$$E = \frac{1}{2L}\int_0^T \{\sum_{i=1}^{2} |q_i(\dot{\psi} - \dot{\alpha}_i)| + |u_2(\dot{\alpha}_2 - \dot{\beta}_2)| \quad (12)$$

$$+ |p_2(\dot{\beta}_2 - \dot{\gamma}_2)|\} dt$$

is used for solving *Problem A.* The objective functional (11) estimates the energy expenditure per unit of distance traveling of ALS [2, 6, 7]. The same approach as described in paragraph 3 has been used for solving the problem of design energy-optimal above-knee prostheses. Due to the dynamic constraints (11) the procedure of converting the *Problem A* into the standard nonlinear programming problem includes the solution of the semi-inverse dynamics problem for the controlled mechanical system that models ALS with above-knee prosthesis.

The computations were carried out for ALS of a person of height 1.76 m, and mass 73.2 kg. The respective values of linear and mass-inertia characteristics of particular links of ALS were calculated on the basis of known experimental data and can be found in [7].

Problem A has been solved numerically for two types of the prostheses: the above-knee prosthesis with the hydraulic actuator at knee and the prosthesis with the pneumatic knee mechanism. For both of these prostheses three types of human gait have been studied, characterized by different values for the duration of a double step *T,* velocity *V,* and length of step *L*: slow walking with T_S=1.383 s, V_S=0.998 m/s, L_S=0.69 m; walking at a normal pace T_N=1.1396 s, V_N=1.325 m/s, L_N=0.755 m; fast walking at T_F=0.9733 s, V_F=1.685 m/s, L_F=0.82 m [7].

The analysis of the solutions obtained has shown that the kinematic, dynamic, and energetic characteristics of controlled motion of ALS are strongly sensitive to the essential prosthesis' parameters. For a given individual and pace of a gait there exist optimal values of parameters of the prosthesis' knee and ankle mechanisms $C_p = (C, K, D, d_1, d_2, l_0, S_p, S_0)$. These parameters give minimum energy expended per unit of distance traveled. For above mentioned types of human walking we obtained the following minimal values for the energy consumption for slow, normal and fast paces of motion respectively: E_S=117 J/m, E_N=114 J/m, E_F=147 J/m (for pneumatic knee mechanism), and E_S=103 J/m, E_N=96 J/m, E_F=125 J/m (for hydraulic knee mechanism). Comparison of these data shows that the normal pace of the ALS gait gives a minimum to the energy expended per unit of distance traveled comparing to the amount of energy needed for the slow or fast gaits. This is valued for both energy-optimal pneumatic and hydraulic knee mechanisms.

Some kinematic and dynamic characteristics of the energy-optimal motion of ALS with optimal structure of the above-knee prosthesis obtained by the numerical solution of *Problem A* for the gait with normal pace are shown in *Figures 3 - 4* (solid thin curves correspond to the prosthesis with the hydraulic actuator at the knee, dashed curves - to the prosthesis with the pneumatic knee mechanism).

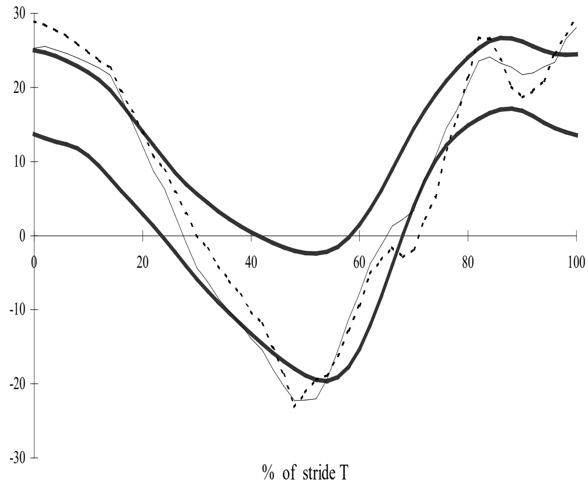

Figure 3. Hip Angle of the Prosthetic Leg, ($\alpha_1 - \psi$), in degrees.

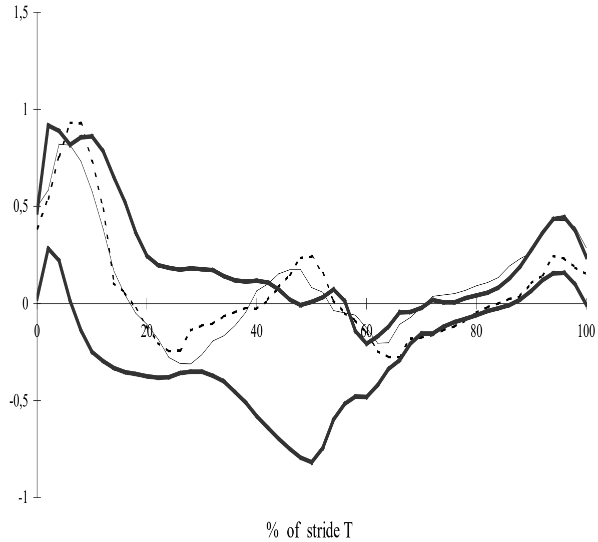

Figure 4. Hip Torque of the Prosthetic Leg, ($q_1(t)/M$), in Nm/kg.

For the comparison purposes in *Figures 3-4* the domains of the values of the respective kinematic and dynamic characteristics obtained by the biomechanical experiments for a human normal gait [7] are depicted by heavy solid curves.

Analysis of the plots that depicted in Figures 3-4 shows that the kinematic and dynamic characteristics of the motion of ALS with obtained energy-optimal structure of above-knee prostheses are within reasonable proximity to the respective characteristics of a human normal gait [7].

6 Conclusion

We have formulated an optimal control problem for semi-passively actuated mechatronic system (*Problem A*). The key feature of the proposed mathematical statement is the direct utilization of the equations describing the inherent dynamics of the passive actuators together with all other constraints imposed on the state vector and the controlling stimuli of the system. It leads to the non-uniqueness of the solution of the direct and inverse dynamics problems and makes it possible to design optimally both structure (passive actuators) and external control of mechatronic system.

The problem of energy-optimal control of overactuated closed-loop chain SCARA-like robot having both powered and unpowered drives has been solved analytically. It has been shown that optimally designed overactuation can decrease the energy consumption needed for arbitrary prescribed motion of the considered semi-passively controlled robot. Moreover, the previous study [3, 5] demonstrated that incorporation of the optimal passive linear spring-damper actuators into the structure of the considered closed-loop chain robot leads to a significant reduction of the energy consumption of the robot for cyclic pick and place operations.

For solving optimization problems for general type of semi-passively actuated mechatronic systems the numerical method has been presented. Efficiency of the proposed method is illustrated by the solution of design problem of the energy-optimal above-knee prostheses with two types of passively controlled knee mechanisms.

Results obtained give some insight into the study of questions about the role of inherent dynamics in controlled motion, and how much mechatronic system should be governed by the external drives and how much by the system's inherent dynamics.

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